Gamma-weighting for unresolved variability, from fluxes through GCM sells to radiances in satellite pixels

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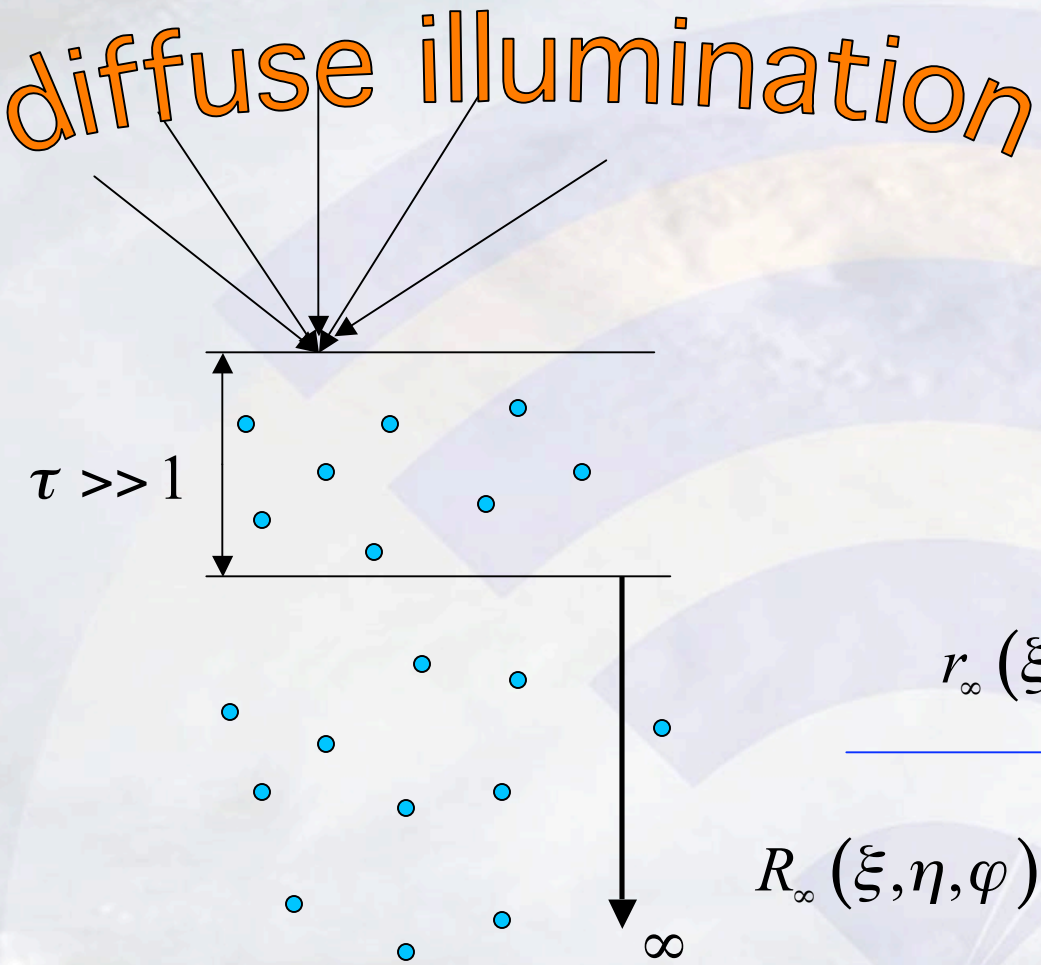
I. Asymptotic theory

Physical derivations

No absorption



$$r_\infty = r(\tau) + t(\tau)$$

$$r_\infty(\xi) = r(\xi,\tau) + t(\tau)u(\xi)$$

$$R_\infty(\xi,\eta,\varphi) = R(\xi,\eta,\varphi,\tau) + t(\tau)u(\xi)u(\eta)$$

$$R_\infty(\xi,\eta,\varphi) = R(\xi,\eta,\varphi,\tau) + t(\tau)u(\xi)u(\eta)$$

$$\downarrow$$

$$R_\infty(\xi,\eta,\varphi) = R(\xi,\eta,\varphi,\tau) + (1 - r(\tau))u(\xi)u(\eta)$$

$$\downarrow$$

$$R_\infty(\xi,\eta,\varphi) = R(\xi,\eta,\varphi,\tau) + (r_\infty - r(\tau))u(\xi)u(\eta)$$

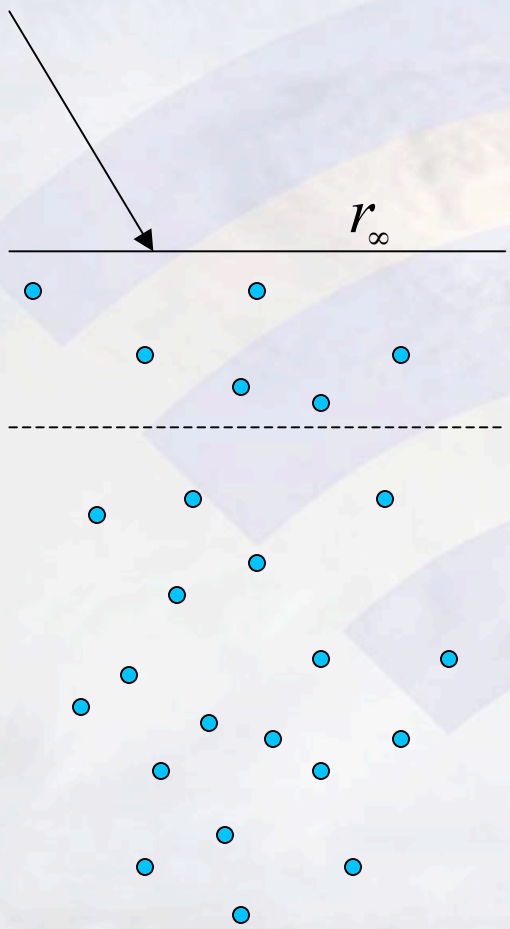
$$T(\xi,\eta) = t(\tau)u(\xi)u(\eta)$$

Weakly absorbing media:

$$R(\xi,\eta,\varphi,\tau) = R_\infty(\xi,\eta,\varphi) - (r_\infty - r(\tau))u(\xi)u(\eta)$$

$$T(\xi,\eta) = t(\tau)u(\xi)u(\eta)$$

Weak absorption



$$r_\infty = r(\tau) + t(\tau) A t(\tau)/(1 - r(\tau) A)$$

$$r_\infty = r(\tau) + \Delta r$$

$$A = r_\infty$$

$$\Delta r = t(\tau) r_\infty e^{-k\tau}$$

$$r = \frac{(1 - e^{-2x}) r_\infty}{1 - r_\infty^2 e^{-2x}} \qquad t = \frac{(1 - r_\infty^2) e^{-x}}{1 - r_\infty^2 e^{-2x}}$$

$$x = k\tau$$

Semi-infinite media

$$r_\infty(\beta) = \sum_{n=1}^{\infty} a_n (1-\beta)^n \qquad r(0) = \sum_{n=1}^{\infty} a_n = 1 \qquad \beta = 1 - \omega_0$$

$$(1-\beta)^n = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \beta^j \qquad \binom{n}{j} \equiv \frac{n!}{j!(n-j)!}$$

$$r_\infty(\beta) = \sum_{n=1}^{\infty} a_n \sum_{j=0}^{n} (-1)^j \binom{n}{j} \beta^j =$$

$$\sum_{n=1}^{\infty} a_n \left[1 - \beta n + \frac{\beta^2 n(n-1)}{2} - \frac{\beta^3 n(n-1)(n-2)}{6} + \ldots \right]$$

$$r_\infty = 1 - \beta\overline{n} + \frac{\beta^2\overline{n(n-1)}}{2} - \frac{\beta^3\overline{n(n-1)(n-2)}}{6} + \ldots$$

$$\overline{n} = \sum_{n=1}^{\infty} n a_n,\ \overline{n(n-1)} = \sum_{n=1}^{\infty} n(n-1)a_n,\ \overline{n(n-1)(n-2)} = \sum_{n=1}^{\infty} n(n-1)(n-2)a_n$$

$$\overline{n(n-1)} \approx \overline{n^2},\ \overline{n(n-1)(n-2)} = \overline{n^3}$$

$$r_\infty = \overline{\exp(-\beta n)} = 1 - \beta\overline{n} + \frac{\beta^2}{2}\overline{n^2} - \frac{\beta^3}{6}\overline{n^3} + \ldots = \exp\left(-\beta\overline{z}\right)$$

The radiative transfer result

$$r_\infty = 1 - 4\sqrt{\frac{\beta}{3(1-g)}}$$

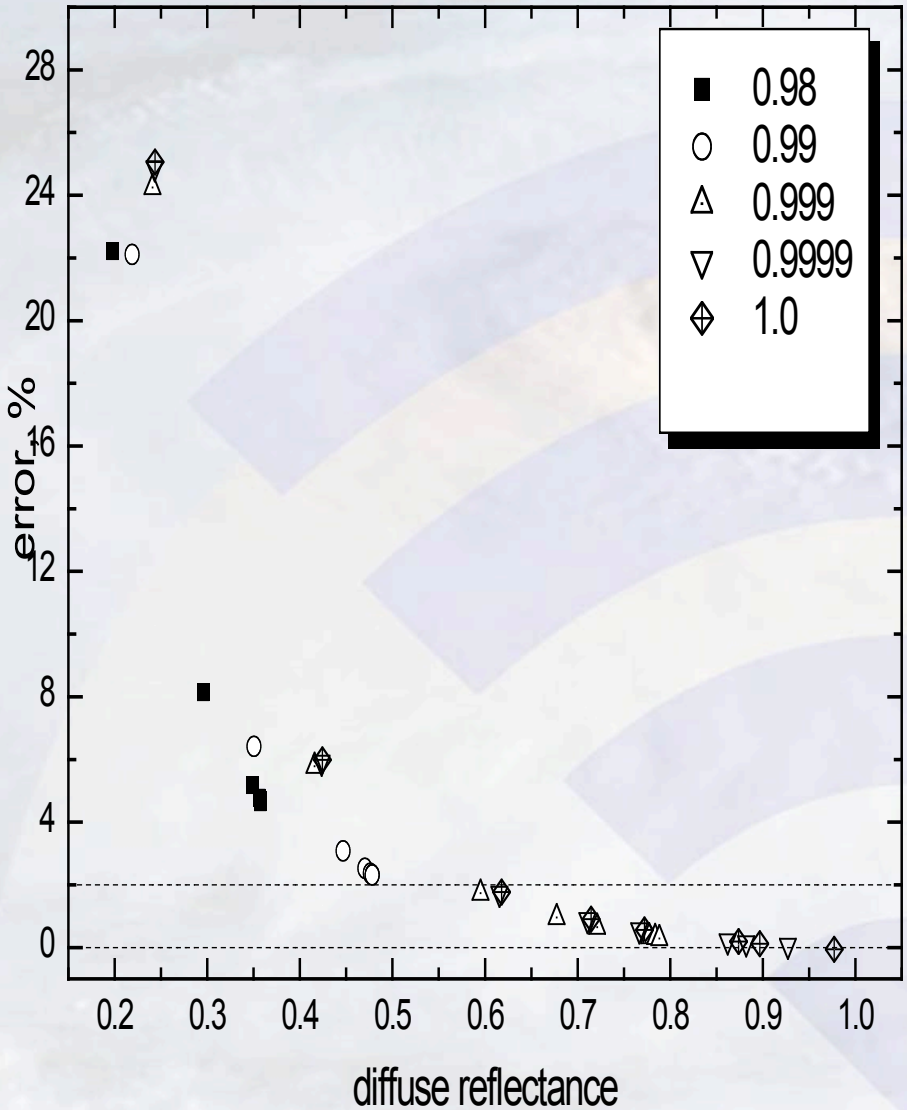
Our equation:

$$r_\infty = \exp\left(-\beta \bar{z}\right) \approx 1 - \beta \bar{z}$$

$$\bar{z} = \frac{4}{k}$$

$$r_\infty = \exp\left(-\frac{4\beta}{k}\right)$$

$$k = \sqrt{3(1-g)\beta}$$



$$r(\tau) = \frac{e^{-y} - e^{-2x-y}}{1 - e^{-2x-2y}}$$

$$t(\tau) = \frac{e^{-x} - e^{-2y-x}}{1 - e^{-2x-2y}}$$

$$x = k\tau$$

$$y = 4\sqrt{\frac{\beta}{3(1-g)}}$$

FINAL RESULTS

$\omega_0 \to 1$ $\tau \to \infty$

$$R(\xi,\eta,\varphi,\tau) = R_\infty(\xi,\eta,\varphi) - (r_\infty - r(\tau))u(\xi)u(\eta)$$

$$r(\xi,\tau) = r_\infty(\xi) - (r_\infty - r(\tau))u(\xi)$$

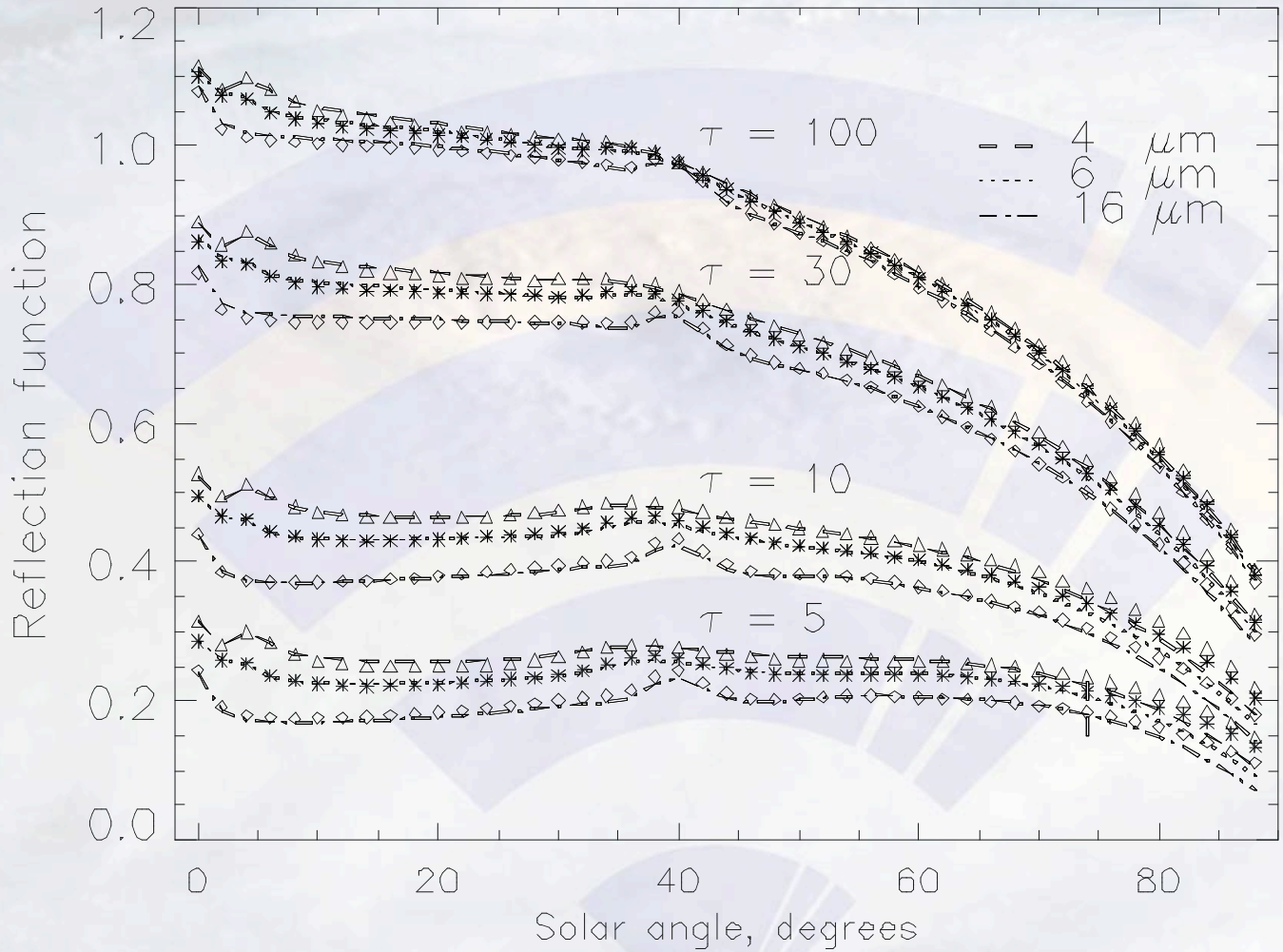
$$T(\xi,\eta,\tau) = t(\tau)u(\xi)u(\eta)$$

$$t(\xi,\tau) = t(\tau)u(\xi)$$

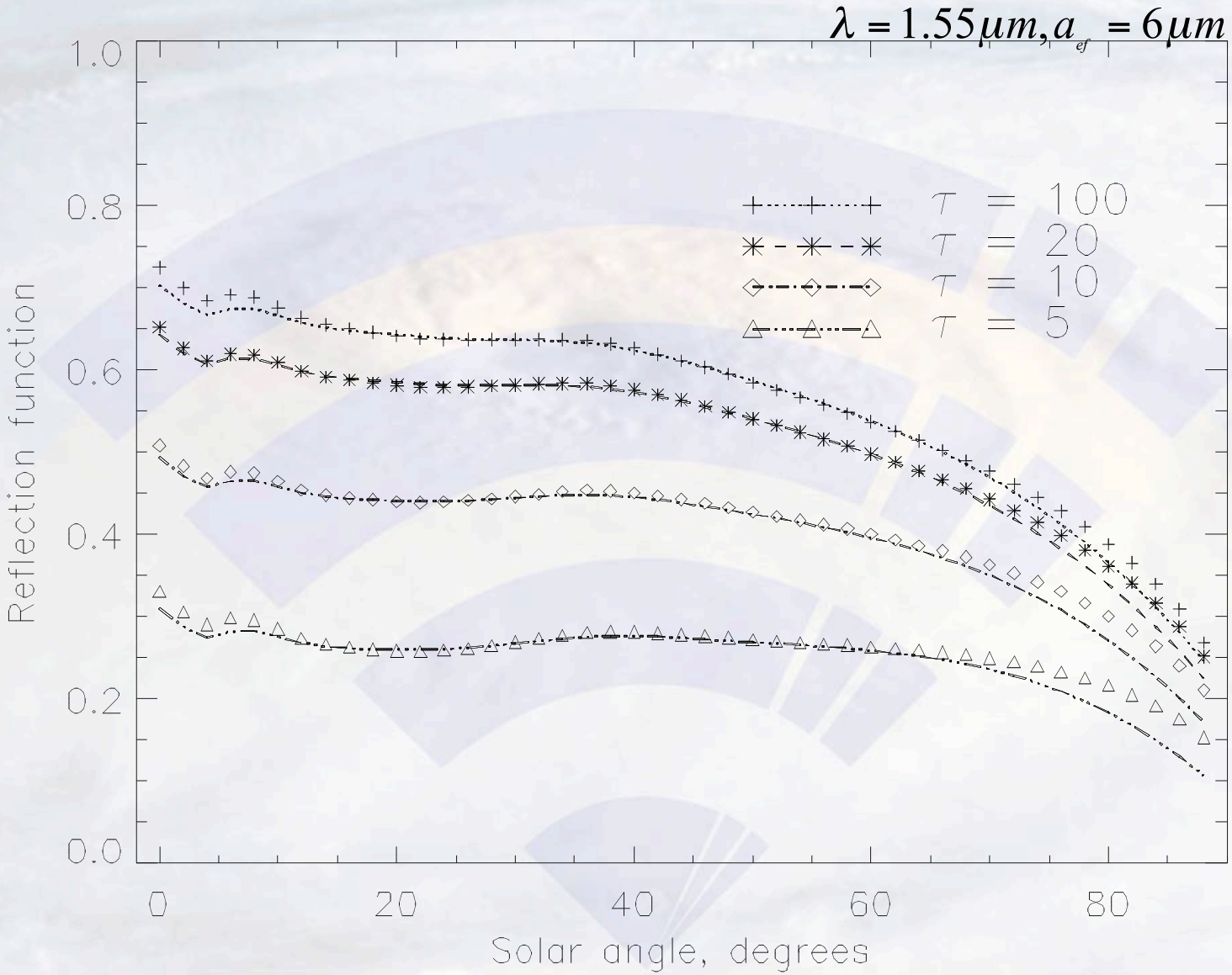
$$u(\xi) = \frac{3}{7}(1+2\xi)$$

The accuracy of the exponential approximation

$\lambda = 0.65 \mu m$



The accuracy of the exponential approximation



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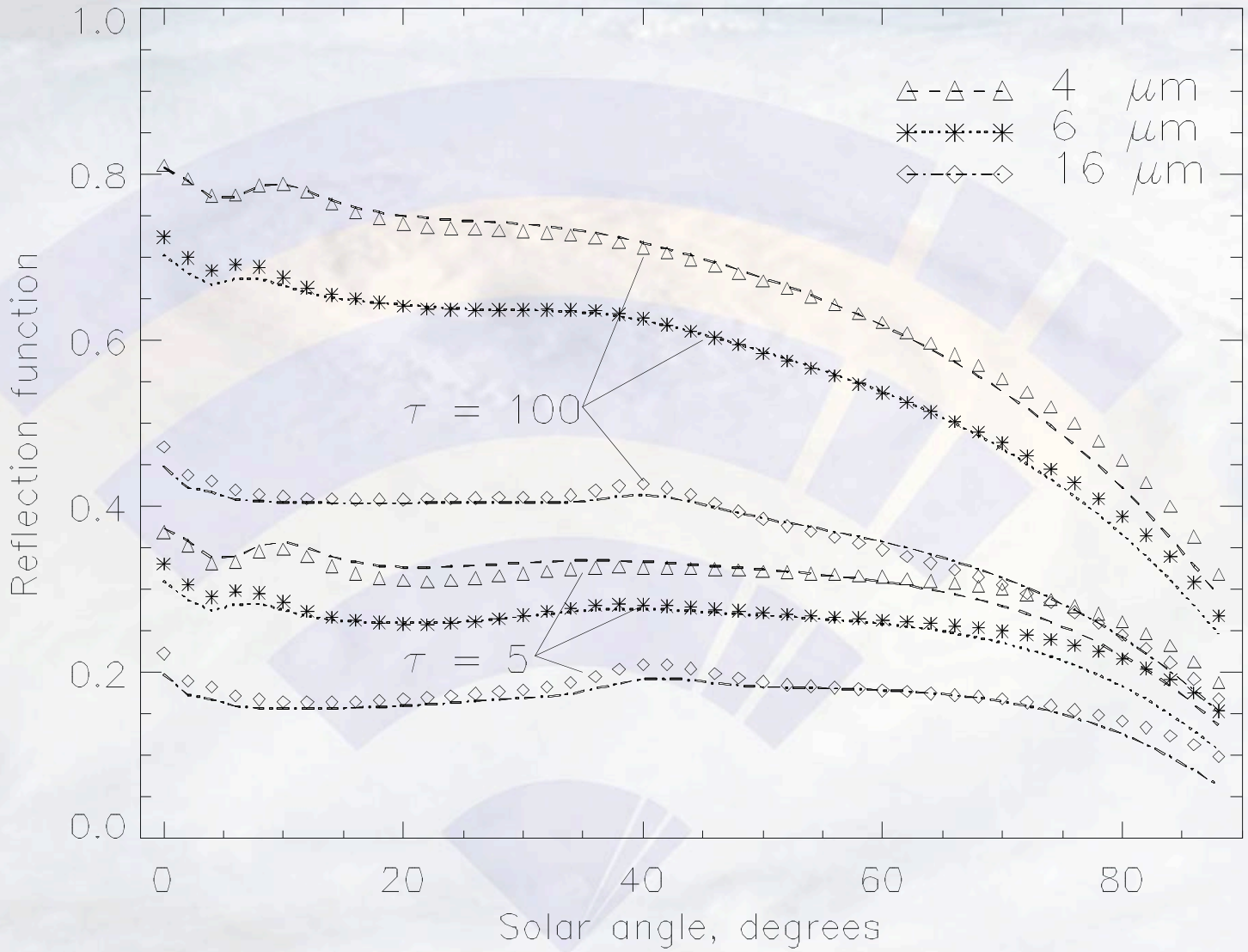
The accuracy of the exponential approximation

$\lambda = 1.55 \mu m$



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II. Relationships between average radiative transfer characteristics

Horizontally inhomogeneous media

$$\bar{R}(\xi,\eta,\varphi) = R_\infty(\xi,\eta,\varphi) - (r_\infty - \bar{r})u(\xi)u(\eta)$$

$$\bar{r}(\xi) = r_\infty(\xi) - (r_\infty - \bar{r})u(\xi)$$

$$\bar{T}(\xi,\eta) = \bar{t}u(\xi)u(\eta)$$

$$\bar{t}(\xi) = \bar{t}u(\xi)$$

$$\delta_T = \delta_{t_d} = \delta_t$$

$$\delta_R = P\delta_r$$

$$\delta_{r_d} = Q\delta_r$$

$$\bar{R} = \int_0^\infty R(\tau)f(\tau)d\tau$$

$$P = \frac{\bar{r}V}{1-(r_\infty - \bar{r})V}$$

$$\delta_R = 1 - \frac{R(\bar{\tau})}{\bar{R}} \quad < 0$$

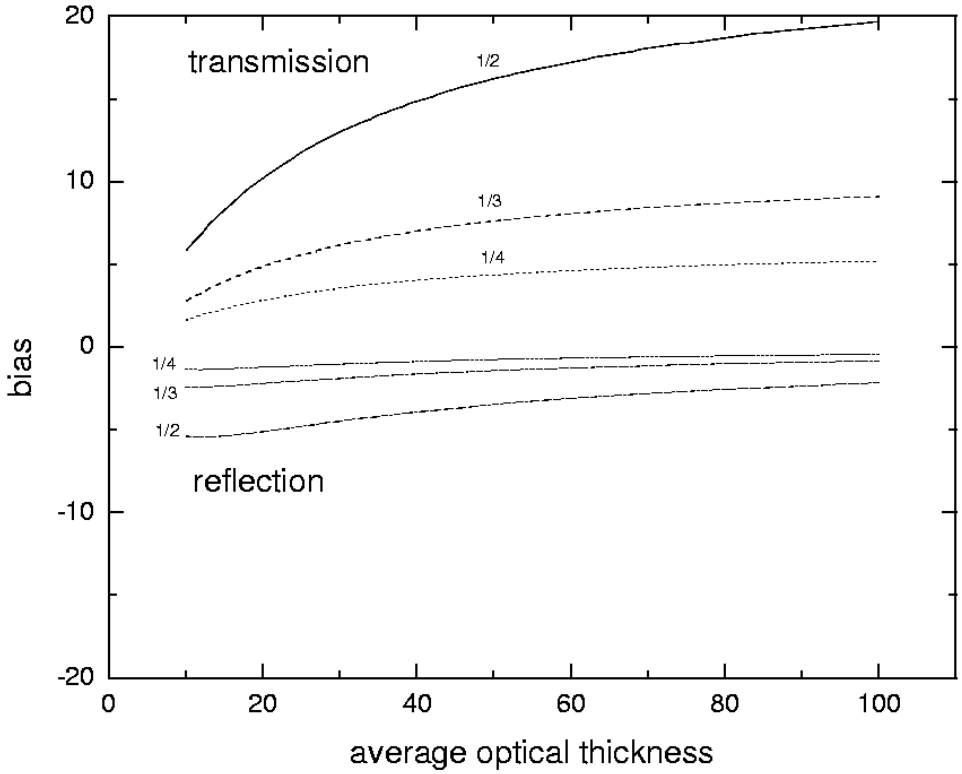
$$Q = \frac{\bar{r}u(\xi)}{1-(r_\infty - \bar{r})V}\delta_r$$

$$\rho_R = \frac{\sigma_R}{\bar{R}}$$

$$\bar{T} > T(\bar{\tau})$$

$$\bar{R} < R(\bar{\tau})$$

$$V = \frac{u(\xi)u(\eta)}{R_{\infty 0}}$$


transmission
1/2
1/3
1/4
1/4
1/3
1/2
reflection
bias
average optical thickness
20
10
0
-10
-20
0
20
40
60
80
100

$$t = \frac{1}{1 + a\tau} \rightarrow (\tau \rightarrow \infty) \rightarrow \frac{1}{a\tau}$$

$$\bar{t} = \frac{1}{a\tau_0}(\tau_0 \rightarrow \infty)$$

$$\delta_t = 1 - \frac{t(\bar{\tau})}{\bar{t}} => 1 - \frac{\tau_0}{\bar{\tau}} = \frac{1}{1+\mu} = \rho_\tau^2$$

$$\delta_t \rightarrow \rho_\tau^2$$

$$as\ \tau_0 \rightarrow \infty$$

$$f(\tau) = A\tau^\mu \exp\left\{-\mu \frac{\tau}{\tau_0}\right\}$$

$$\rho_\tau = \frac{\sigma_\tau}{\bar{\tau}}$$

III. Relationships between statistical distributions

$$t(\tau) = \frac{1}{1 + a\tau}$$

$$f_t(t) \longleftrightarrow f_\tau(\tau)$$

$$f_Y(Y) = \left|\frac{d\tau}{dY}\right| f_\tau(\tau)$$

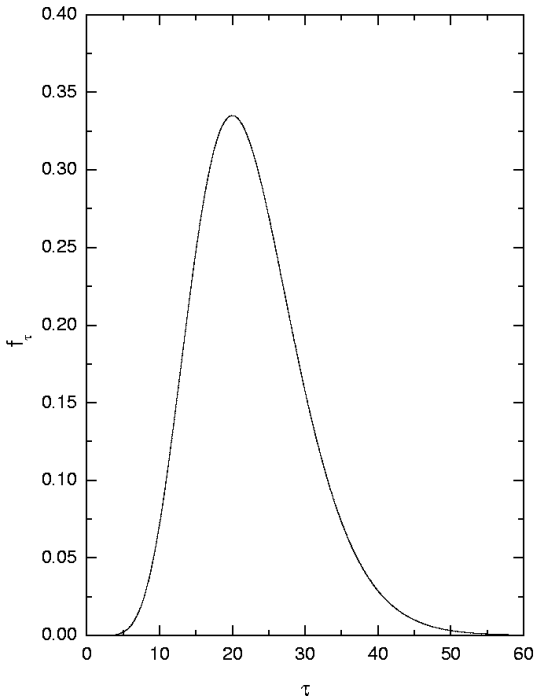
$$f_\tau(\tau) = f_t(t) \Big/ \left|\frac{d\tau}{dt}\right|$$

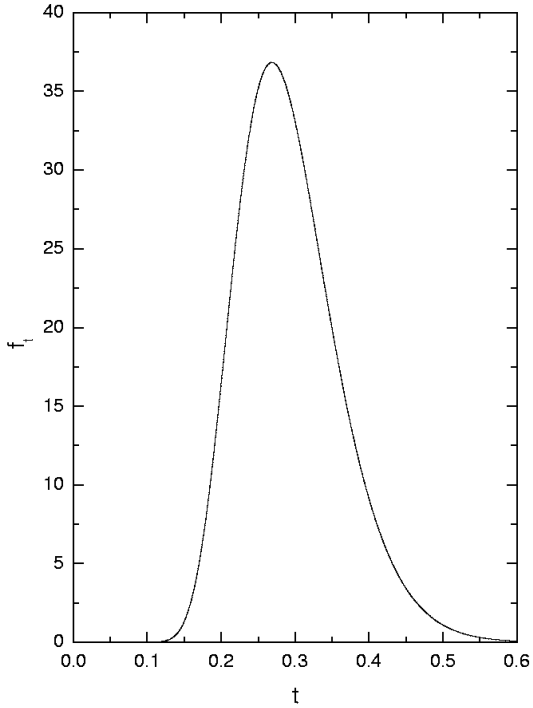
Transmission

$$f_\tau(\tau) = at^2 f_t(t)$$

$$f_\tau(\tau) = at^2 u^{-1}(\xi) u^{-1}(\eta) f_T(T)$$

$$f_\tau(\tau) = at^2 u^{-1}(\eta) f_{t_d}(t_d)$$


0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
f_τ
0
10
20
30
40
50
60
τ


40
35
30
25
20
15
10
5
0
f_t
0.0
0.1
0.2
0.3
0.4
0.5
0.6
t

Arbitrary absorption: optically thick layers

$$R(\tau) = R_\infty - T(\tau) N e^{-k\tau}$$

$$T(\tau) = t(\tau) C^{-2} K(\xi) K(\eta)$$

Germogenova, 1961

$$t(\tau) = \frac{M C^2 e^{-k\tau}}{1 - N^2 e^{-2k\tau}}$$

Derivatives:

Kokhanovsky, 2005, JOSA_A

CONCLUSIONS

- Biases of various radiative radiative transfer characteristics of horizontally inhomogeneous media are interrelated. Asymptotic theory allows to find correspondent analytical relationships for optically thick turbid layers.
- Measured statistical distributions of cloud macroscopic characteristics can be used to deduce statistical distributions of cloud optical thickness

Acknowledgements

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